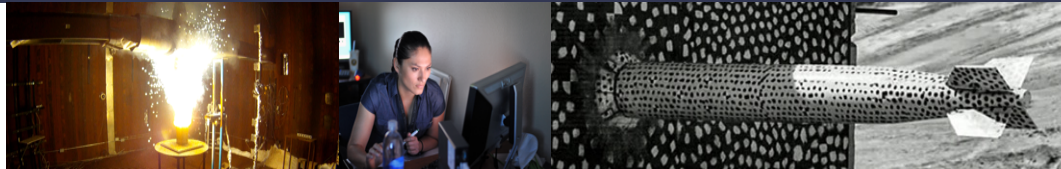


# Explaining Neural Network Predictions for Functional Data Using Principal Component Analysis and Feature Importance



*Talk for ISU Graphics Group*

*September 24, 2020*

**Katherine Goode**

Intern with the Department of Statistical Sciences Sandia National Laboratories



# Talk Overview

1. Internship with Sandia National Labs
2. Project Objectives
3. Our Approach
4. Background on FPCA and PFI
5. Application and Visualizations
6. Going Forward





# Sandia National Labs

A brief overview and summary of my internship experience

# Sandia National Labs



*National security is our business. We apply science to help detect, repel, defeat, or mitigate threats.*

For more than [70 years](#), Sandia has delivered essential science and technology to resolve the nation's most challenging security issues.

Sandia National Laboratories is operated and managed by National Technology and Engineering Solutions of Sandia, LLC., a wholly owned subsidiary of Honeywell International, Inc. National Technology and Engineering Solutions of Sandia operates Sandia National Laboratories as a [contractor](#) for the U.S. Department of Energy's National Nuclear Security Administration (NNSA) and supports numerous federal, state, and local government agencies, companies, and organizations.

As a Federally Funded Research and Development Center ( [FFRDC](#)), Sandia may perform work for industry responding to certain types of federal government solicitations. The solicitation must allow FFRDC participation and meet the requirements of Sandia's management and operating contract with DOE/NNSA.

## National security programs

We work with other government agencies, industry, and academic institutions to accomplish our missions in the following strategic areas:

- [Nuclear Weapons](#)
- [National Security Programs](#)
- [Energy](#)
- [Global Security](#)



### Computer modeling to understand, track and prepare for COVID-19

National models of COVID-19's spread may be of limited use to state and local planners because the models generally average population data instead of considering the distinctive character of smaller regions. Sandia Labs researchers are using regional data provided by the State of New Mexico to determine the most effective regional distribution of medical supplies and resources and where pop-up testing centers would be best located. Sandia researchers have applied these techniques to other states and localities across the United States.

[Continue Reading](#)



### Testing for COVID-19

To protect mission-essential functions, keep our workforce safe and help reduce the spread of COVID-19 at Sandia facilities, Sandia became the first Department of Energy lab on April 7 to start testing its workforce for COVID-19 infection. The Labs continues to offer on-site swabbing to collect samples for COVID-19 diagnosis of employees and on-site subcontractors who are symptomatic or have come into contact with a presumed or COVID-19 symptomatic case. Swabbing is also provided to employees to meet travel requirements and personnel needing to be...

[Continue Reading](#)

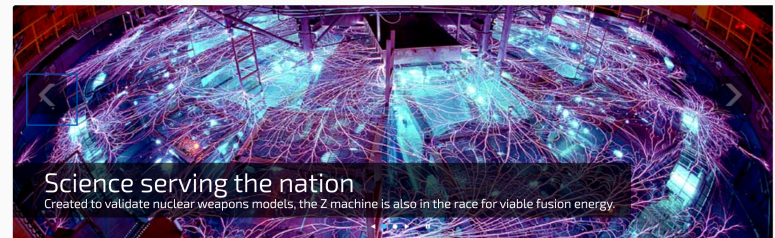


### Manufacturing R&D for personal protective equipment, supplies

Sandia Labs has joined a coalition of national labs aiming to help ensure the nation has access to COVID-19 medical supplies. The coordinated effort is using innovative engineering and technology, such as 3D printing to bridge supply-chain gaps in masks, ventilators and one-time use supplies, such as testing swabs. Examples of this effort are Sandia's method of converting hospital-grade BIPAP machines into ventilators and the labs' help in assessing new decontamination processes to enable the reuse of N95 masks. With an eye to the future, the coalition is...

[Continue Reading](#)

## Z Pulsed Power Facility



### About Z

Sandia's Z machine is the world's most powerful and efficient laboratory radiation source. It uses high magnetic fields associated with high electrical currents to produce high temperatures, high pressures, and powerful X-rays for research in [high energy density science](#). The **Z machine creates conditions found nowhere else on Earth**. Z is part of Sandia's [Pulsed Power](#) program, which began in the 1960s.

### News

- [Four Sandia researchers win Presidential Early Career Award](#)  
August 1, 2019
- [Three Sandia researchers elected fellows of](#)





# Internship (expectations)





# Internship (reality)





# In all seriousness...

- A really good experience
- Working on research with Danny Ries (PhD in statistics from ISU) and other statisticians, computer scientists, and scientists
- In a group of summer group of mathematics, statistics, and computer science interns
- Able to attend seminars from employees at Sandia



# Project Objectives

Predicting explosive device characteristics with machine learning

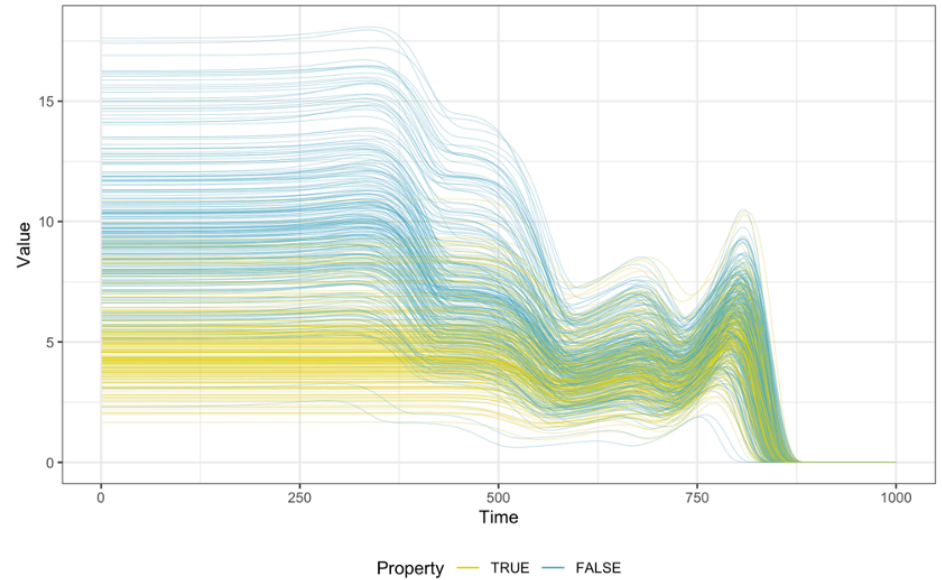


# Main Objective

Use machine learning to predict characteristics of explosive devices based on optical spectral-temporal signatures of explosions



Example Spectral Temporal Signatures

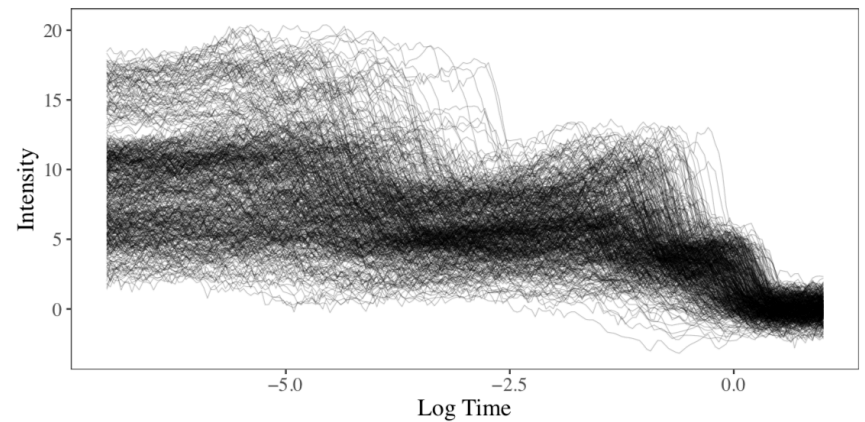
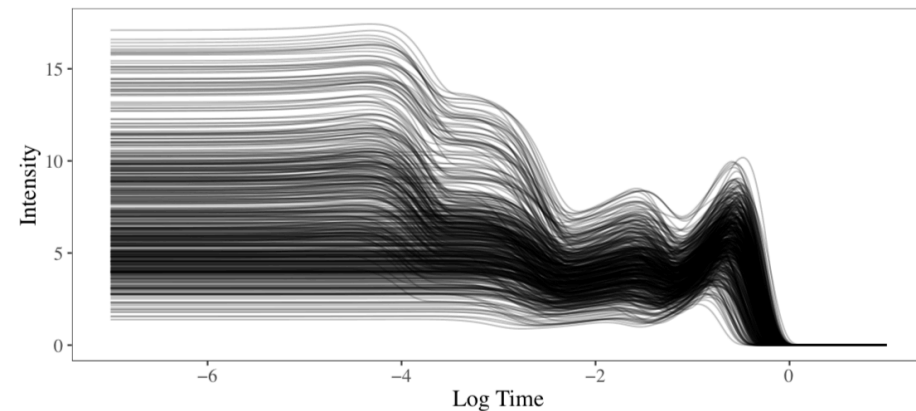


# Sub-Objectives

- Return uncertainty quantification
  - Provide an estimate of uncertainty (such as a confidence interval)
  - Use methods such as Bayesian neural networks (BNNs)
- Provide explanations for model predictions
  - Important for providing trust in the model
  - Motivate use of machine learning in this application to decision makers

# Data

- 10,000 simulated signatures
- Generated based on scientific understanding of relationship between shape of optical signature and three explosive device characteristics
- Simple and degraded versions of the data
- Example signatures:



# Response Variables



## Explosive Device Characteristics

Y1:

- Binary variable
- Affects intensity of signature early in time and timing of first peak

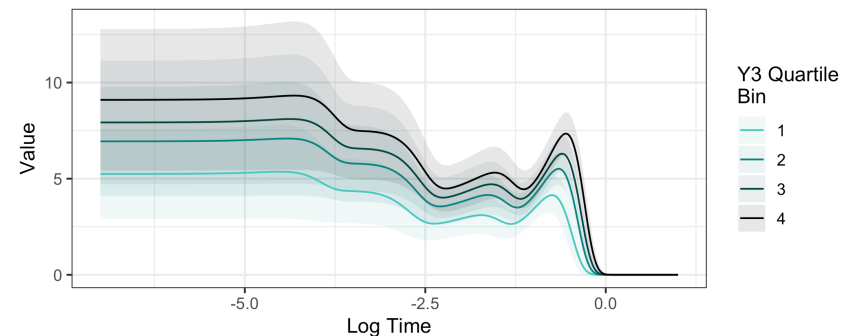
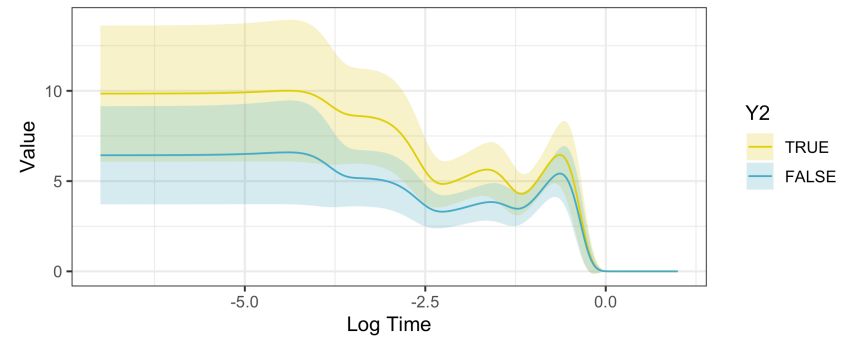
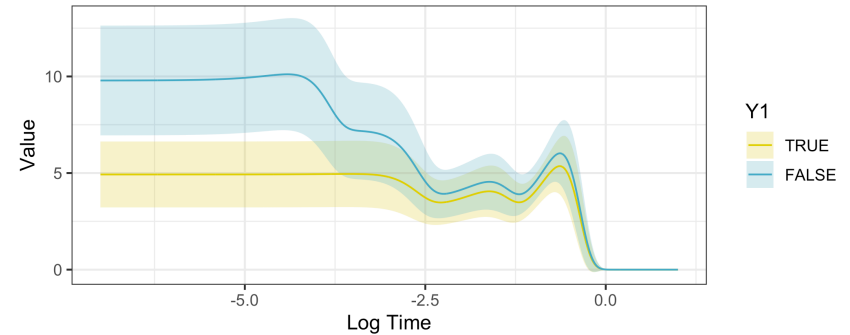
Y2:

- Binary variable
- Affects intensity of signature over all times

Y3:

- Continuous variable
- Affects intensity and timing of all peaks

Pointwise Functional Means of Response Categories  
+/- 1 Pointwise Standard Deviation





# Our Approach

Functional PCA and Permutation Feature Importance

# Naïve Approach

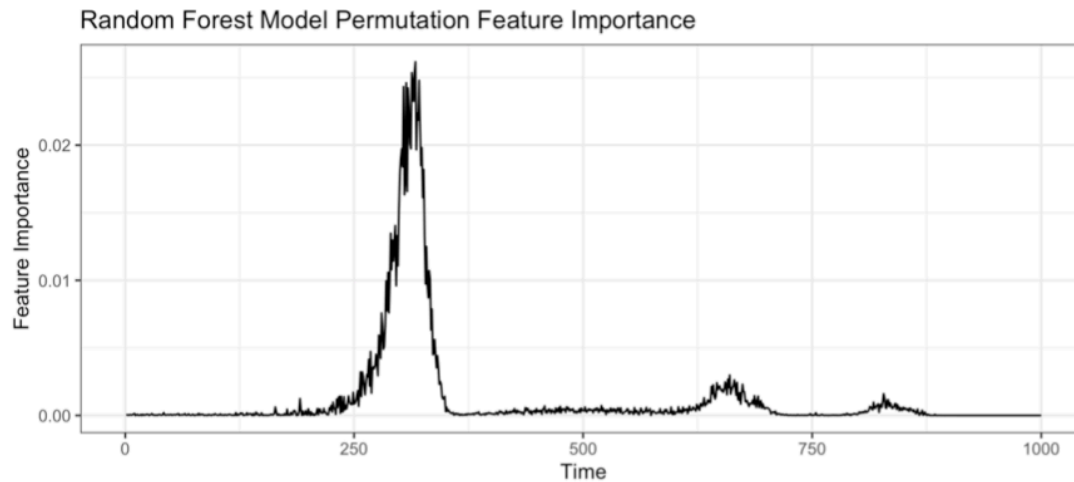
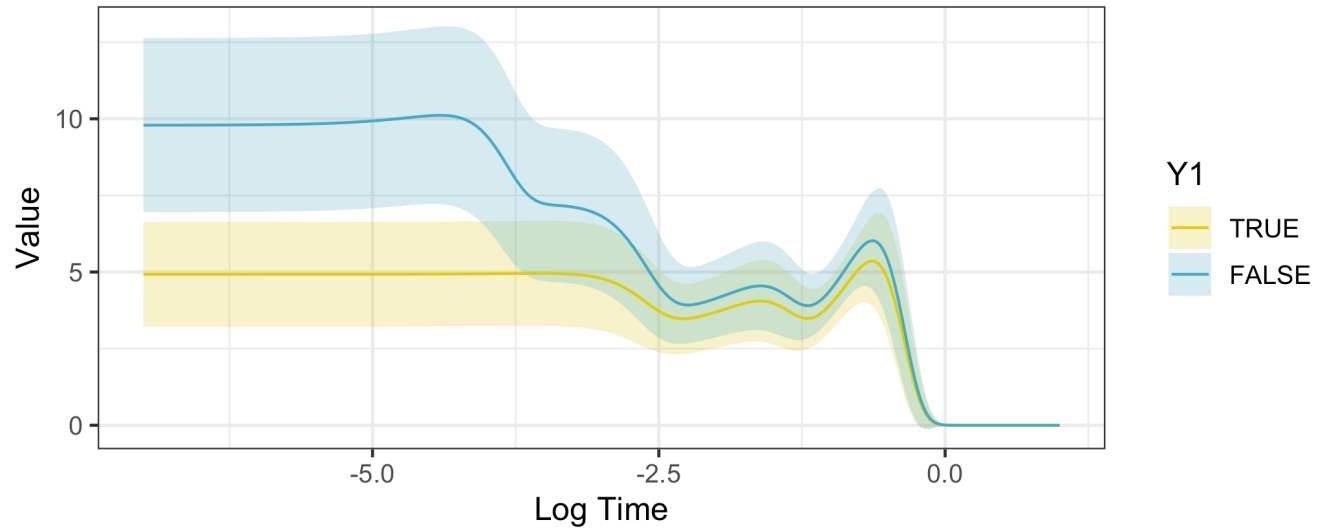
- Use each signature time point as a predictor variable in the model
- Train a random forest, neural network, etc.
- Compute variable importance

Signature	Time 1	Time 2	.....	Time 1000	Y1	Y2	Y3
1	10	9.8	...	0	T	T	82
2	15	15.1	...	0	F	T	71
...	...	...	...	...	...	...	...
10000	8	8.05	...	0	T	F	68

Hypothetical Dataset



# Example Feature Importance for Y1



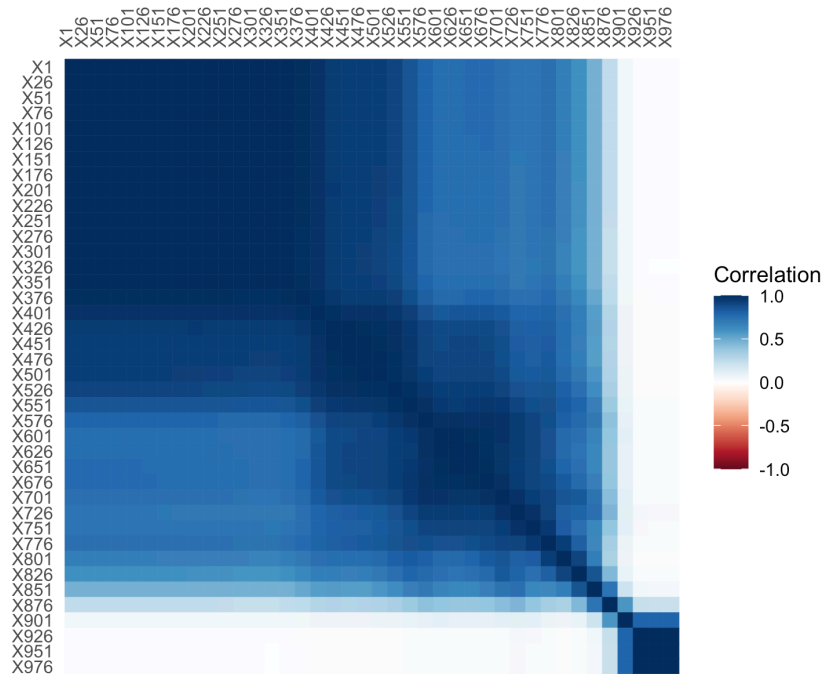


# A problem....

The statistician asks:

*“What about the dependence in the signatures over time?”*

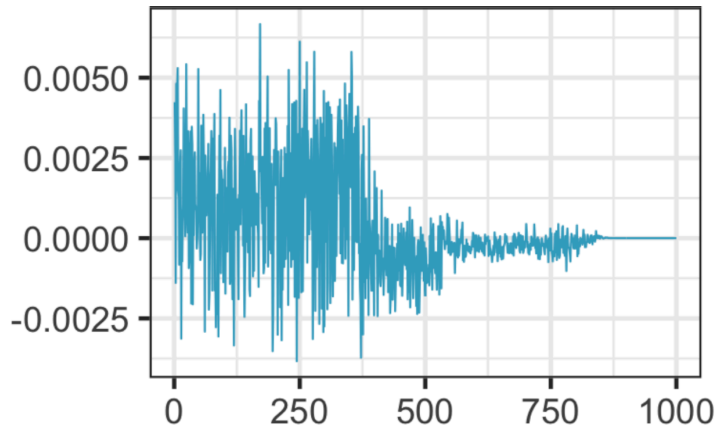
Pairwise Correlations of Every 25 Features



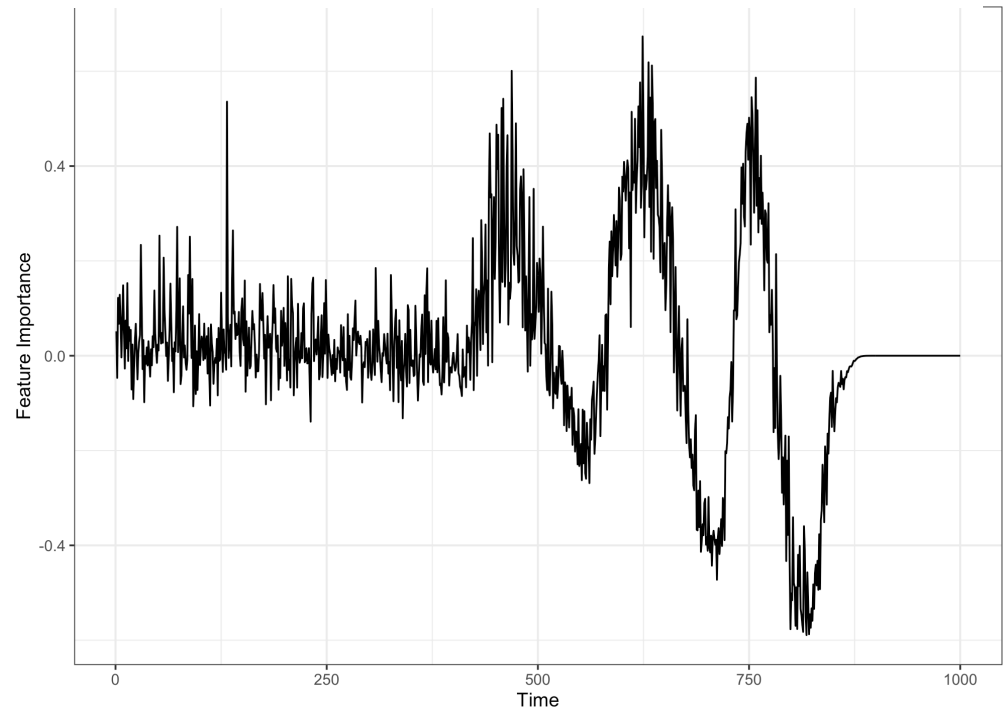
# Neural network feature importance

(via the naïve approach)

Permutation Feature Importance for Y1



Permutation Feature Importance for Y3



# Our Approach

1. Transform signatures using FPCA (creates uncorrelated features)
2. Fit neural network using principal components (PCs) as features
3. Use permutation feature importance to determine important PCs
4. Interpret PCs using visualizations

*Note: Each of the statistical methods mentioned in the above steps have been developed previously. Our novel approach is using these tools in combination to provide an explanation for neural network predictions when working with functional data.*

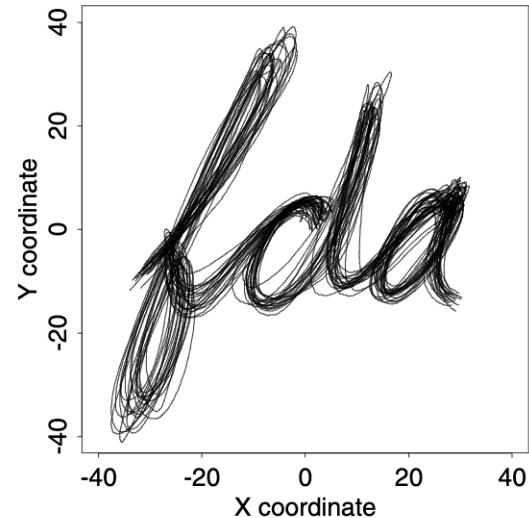
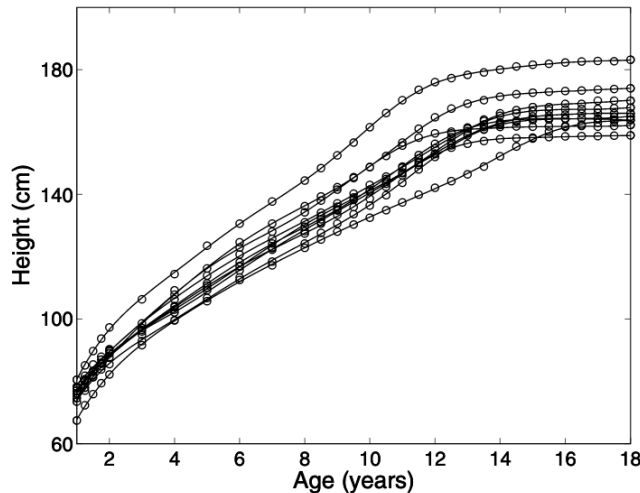


# Background on Statistical Methods

Functional Principal Components and Permutation Feature Importance

# Functional Data Analysis (FDA)

- [Wang, Chiou, and Müller \(2015\)](#) define:
  - “Functional data analysis (FDA) deals with the analysis and theory of data that are in the form of functions, images and shapes, or more general objects”



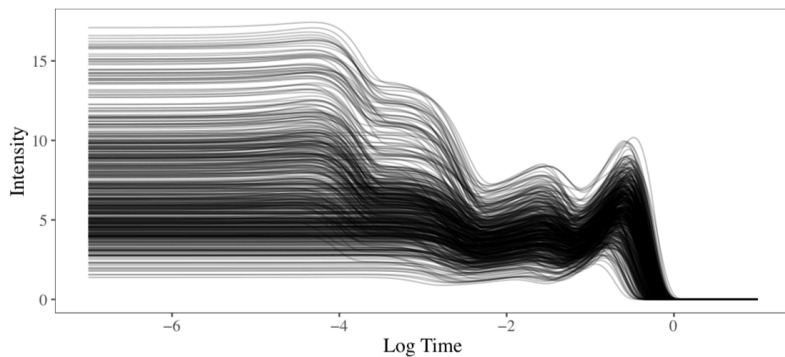
# Functional Principal Components Analysis (FPCA)

- FPCA is essentially PCA with "functional data"
- Transforms original functions in a way that provides nice properties:
  - Independent features
  - First few features capture majority of the variation in original data
- Use first few transformed features to fit model

# Ways to implement FPCA

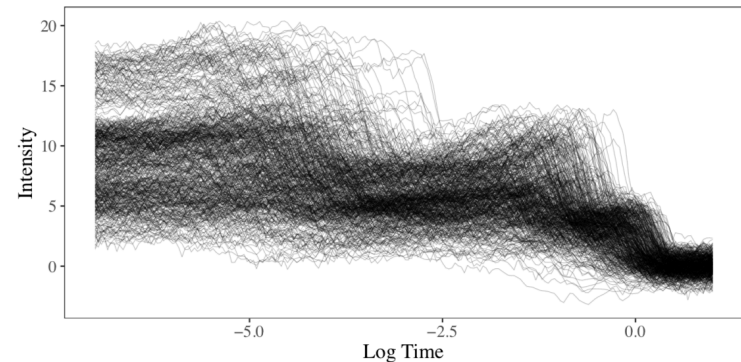
## Basic Method:

- Treat each time as a variable and apply basic PCA
- Accounts for vertical variability



## More Sophisticated Method:

- Joint FPCA
  - [Tucker, Wu, and Srivastava \(2012\)](#)
  - [Sungwon Lee and Sungkyu Jung \(2017\)](#)
- Accounts for vertical and horizontal variability







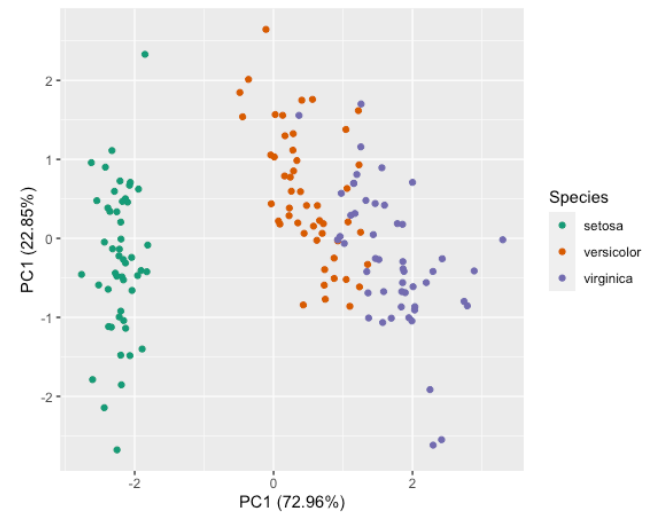
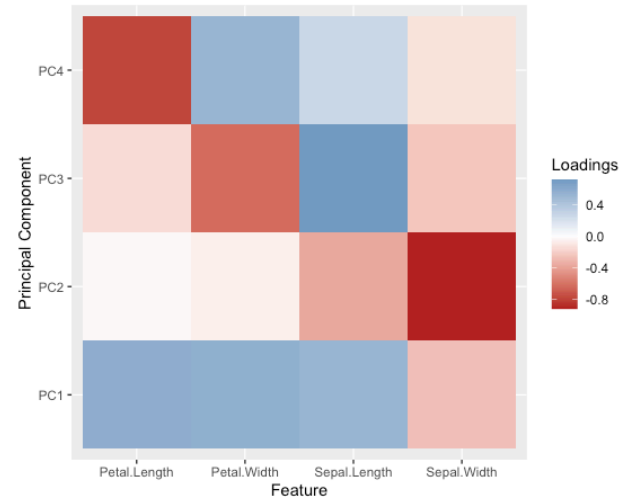
# Interpreting and Visualizing PCA

Applying PCA to iris data:

```
> head(iris)
  Sepal.Length Sepal.Width Petal.Length Petal.Width Species
1         5.1         3.5         1.4         0.2  setosa
2         4.9         3.0         1.4         0.2  setosa
3         4.7         3.2         1.3         0.2  setosa
4         4.6         3.1         1.5         0.2  setosa
5         5.0         3.6         1.4         0.2  setosa
6         5.4         3.9         1.7         0.4  setosa

> pca = prcomp(iris[,-5], scale = TRUE)
> head(pca$x)
      PC1      PC2      PC3      PC4
[1,] -2.257141 -0.4784238  0.12727962  0.024087508
[2,] -2.074013  0.6718827  0.23382552  0.102662845
[3,] -2.356335  0.3407664 -0.04405390  0.028282305
[4,] -2.291707  0.5953999 -0.09098530 -0.065735340
[5,] -2.381863 -0.6446757 -0.01568565 -0.035802870
[6,] -2.068701 -1.4842053 -0.02687825  0.006586116

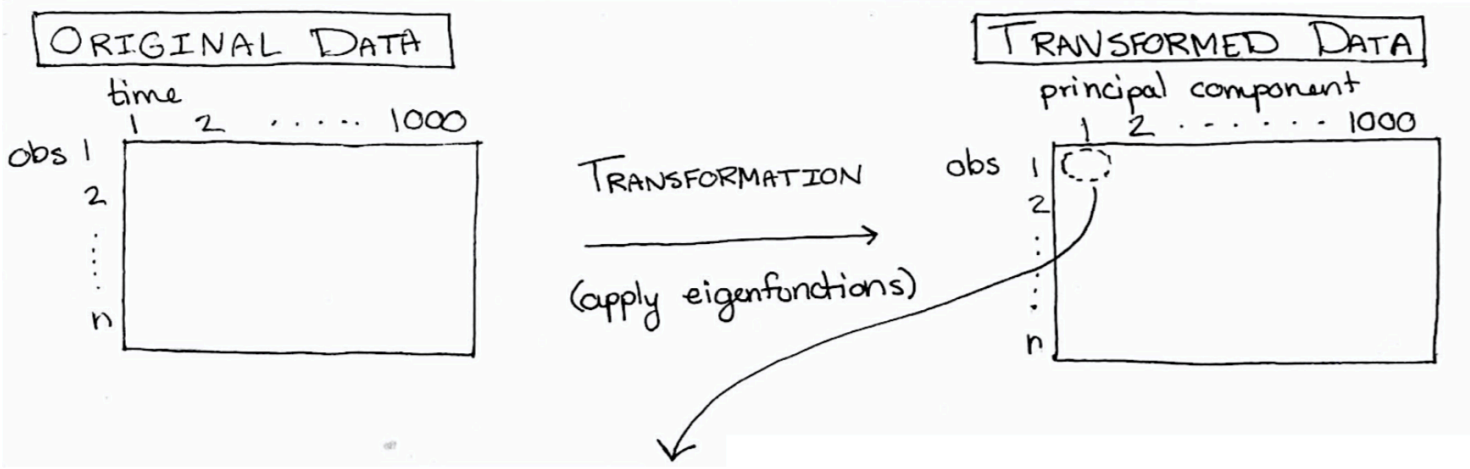
> pca$rotation
      PC1      PC2      PC3      PC4
Sepal.Length  0.5210659 -0.37741762  0.7195664  0.2612863
Sepal.Width  -0.2693474 -0.92329566 -0.2443818 -0.1235096
Petal.Length  0.5804131 -0.02449161 -0.1421264 -0.8014492
Petal.Width  0.5648565 -0.06694199 -0.6342727  0.5235971
```



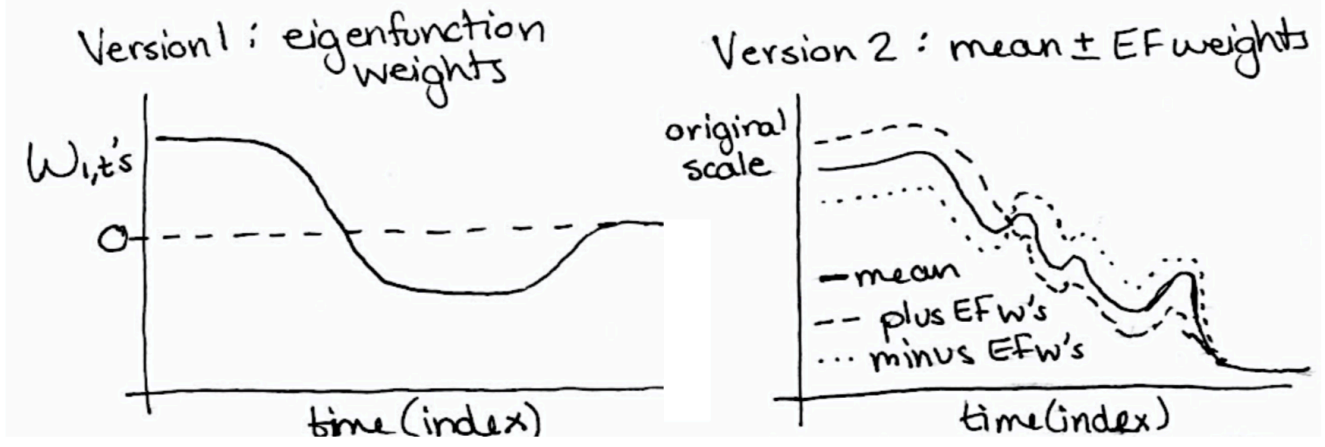


# Visualizing FPCA

**Eigenfunctions:** Functions that transform the original data in FPCA



**Eigenfunction 1:**  $EF1(\text{obs } 1) = (w_{1,t1})(\text{obs}_{1,t1}) + (w_{1,2})(\text{obs}_{1,t2}) + \dots + (w_{1,1000})(\text{obs}_{1,t1000})$





# Visualizing FPCA

Example: Canadian weather temperatures measured over time

## Visualizations of eigenfunctions

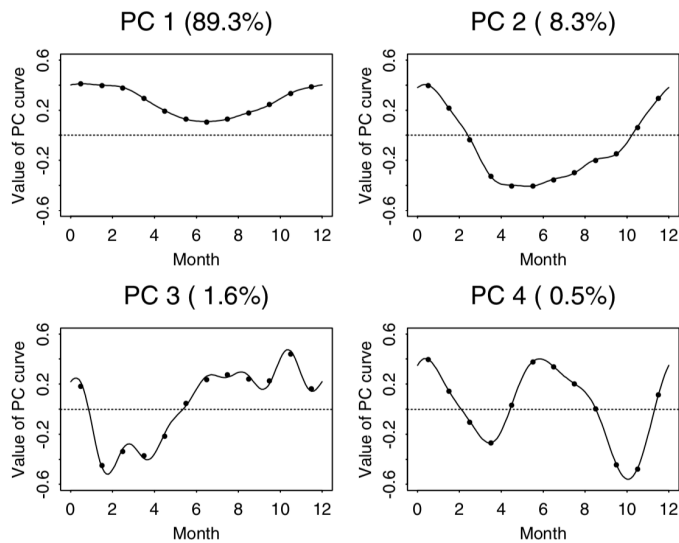


Figure 8.1. The first four principal component curves of the Canadian temperature data estimated by two techniques. The points are the estimates from the discretization approach, and the curves are the estimates from the expansion of the data in terms of a 12-term Fourier series. The percentages indicate the amount of total variation accounted for by each principal component.

## Functional mean +/- eigenfunction weights

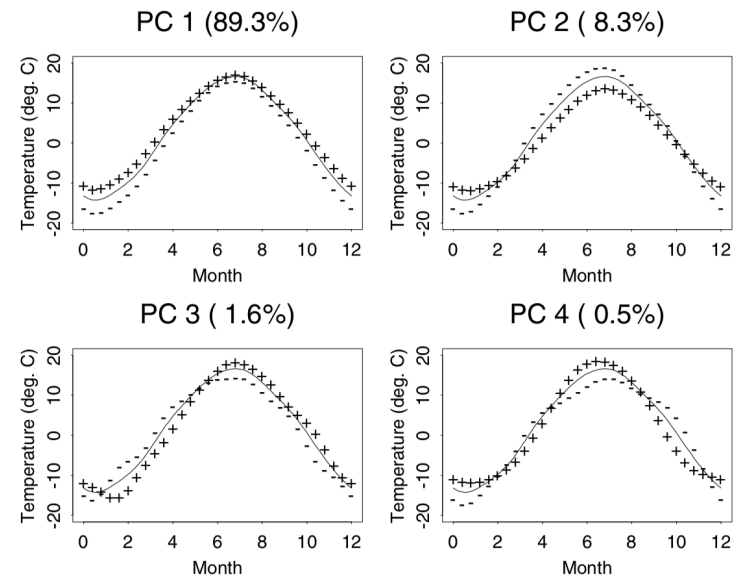
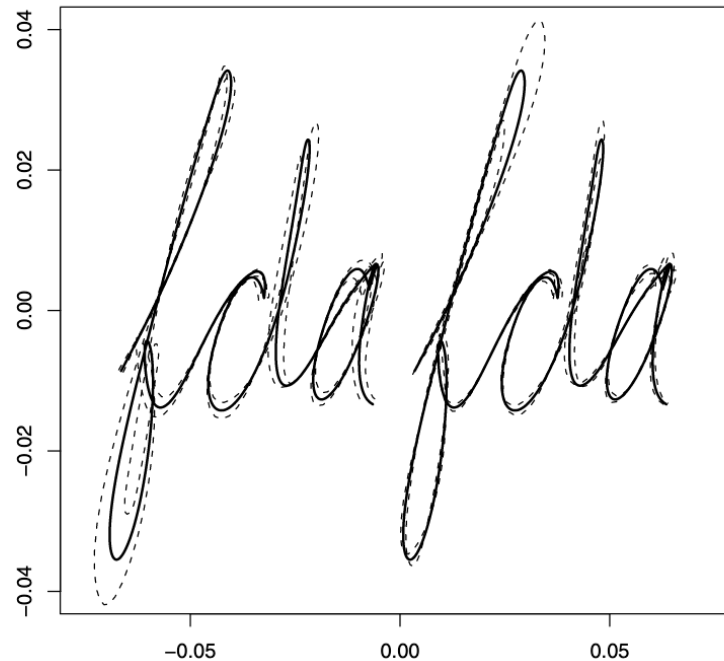


Figure 8.2. The mean temperature curves and the effects of adding (+) and subtracting (-) a suitable multiple of each PC curve.

# Visualizing FPCA (multivariate case)



**Fig. 7.7** Two of the rotated harmonics are plotted as a perturbations of the mean “fda” script, shown as a heavy solid line.

# Permutation Feature Importance (PFI)

- Originally developed by Breiman (2001; [paper](#)) for random forests
- Breiman developed two types of importance:
  - Node impurity based method
  - Permutation based method

```

> library(randomForest)
> set.seed(71)
> iris.rf <- randomForest(Species ~ ., data=iris, importance=TRUE,
+                          proximity=TRUE)
> round(importance(iris.rf), 2)

```

	setosa	versicolor	virginica	MeanDecreaseAccuracy	MeanDecreaseGini
Sepal.Length	5.88	5.87	9.21	10.62	9.37
Sepal.Width	5.23	0.31	4.71	4.94	2.45
Petal.Length	21.60	31.41	27.71	32.39	42.13
Petal.Width	22.96	33.74	32.07	33.85	45.28

- Generalized random forest version for all by Fisher, Rudin, and Dominici (2019; [paper](#))

# PFI: General Idea

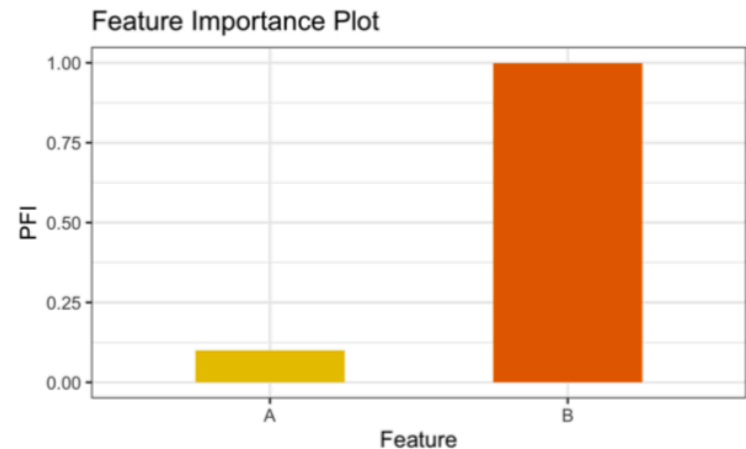
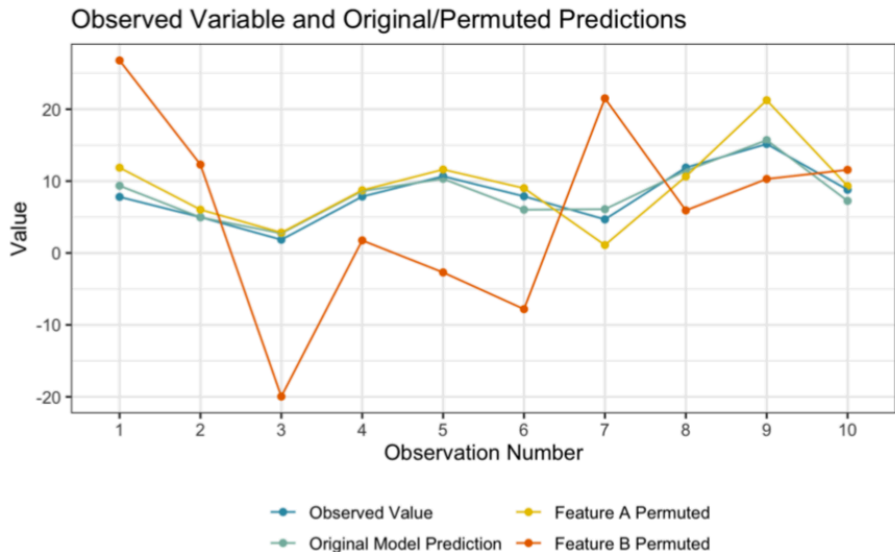


## Procedure

- Permute a feature
- Determine how model predictions are affected
- Repeat for all other features
- Repeat to account for random variation

## Interpretation

- Positive PFI: Prediction affected
- PFI near 0: Prediction NOT affected
- Negative PFI
  - Prediction improved
  - Feature is counterproductive



# PFI: Formula

Define:

- $f$  predictive model
- $y$  response vector
- $X$  training matrix of feature values (rows  $x_i$ )
- $X^{\pi,j}$  matrix with feature  $j$  of  $X$  permuted (rows  $x_i^{\pi,j}$ )
- $L(y_i, f(x_i))$  loss for predicting  $y_i$  from  $f(x_i)$

Permutation Feature Importance:

$$VI_j^\pi = \sum_{i=1}^N L(y_i, f(x_i^{\pi,j})) - L(y_i, f(x_i))$$

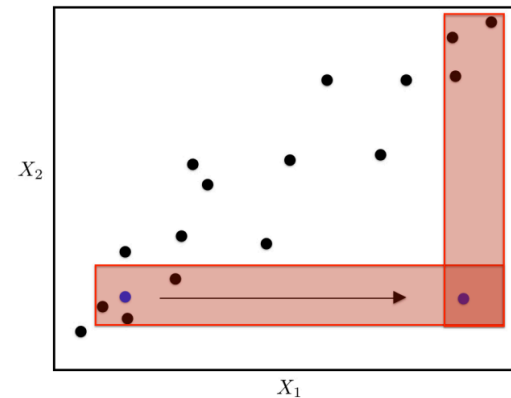
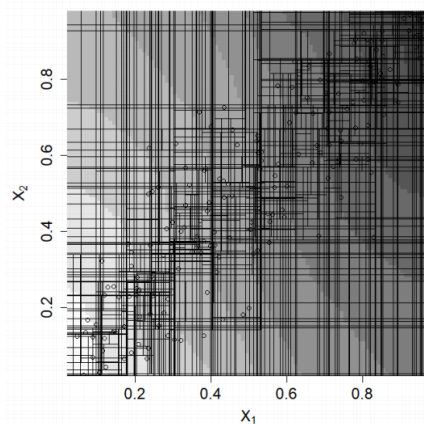
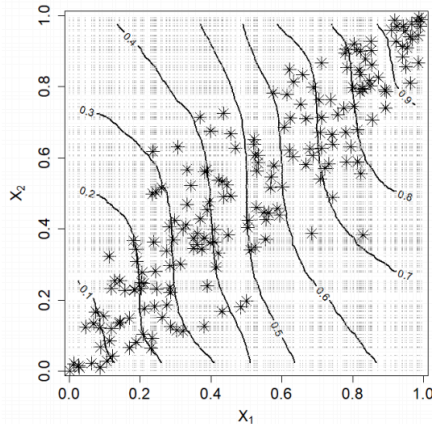


# PFI and Correlation

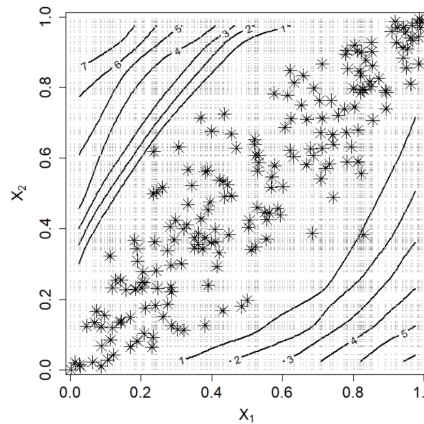
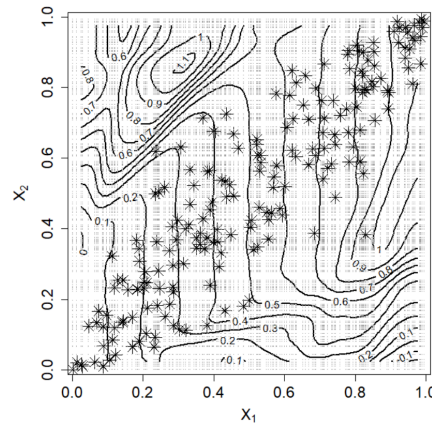


- Research has shown that PFI is biased when correlated variables are involved (Strobl et al. (2007), Archer and Kimes (2008), Nicodemus et al. (2010))
- [Hooker and Mentch \(2019\)](#) attempt to provide an explanation

Random forest



Neural network



[Images from Hooker and Mentch \(2019\)](#)



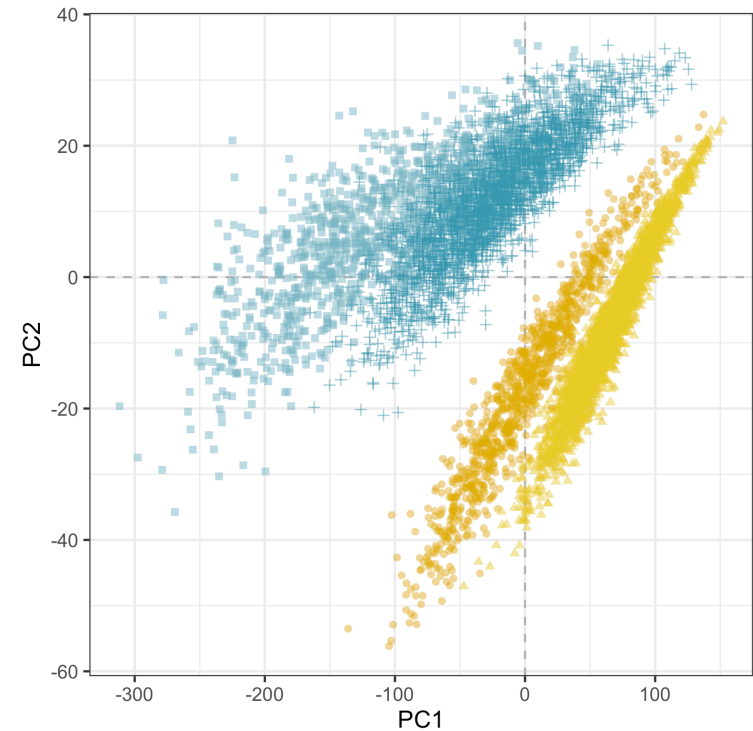
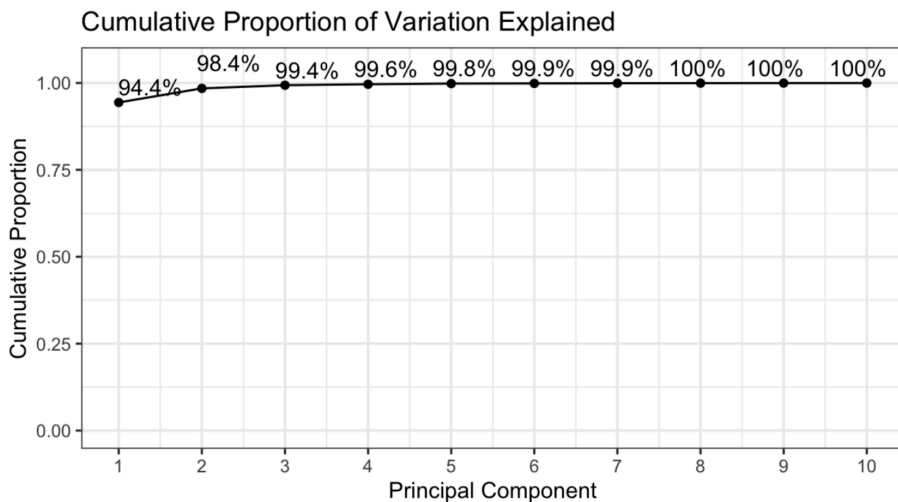
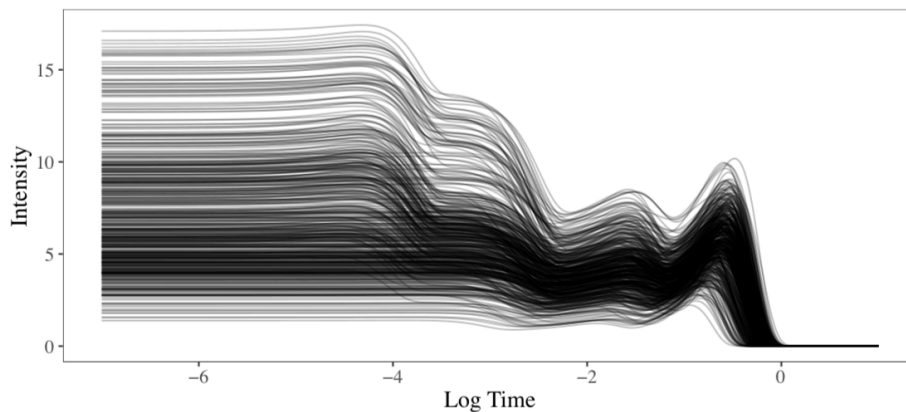
# Application and Visualizations

Explaining neural network predictions



# Step 1: Transform signatures using FPCA

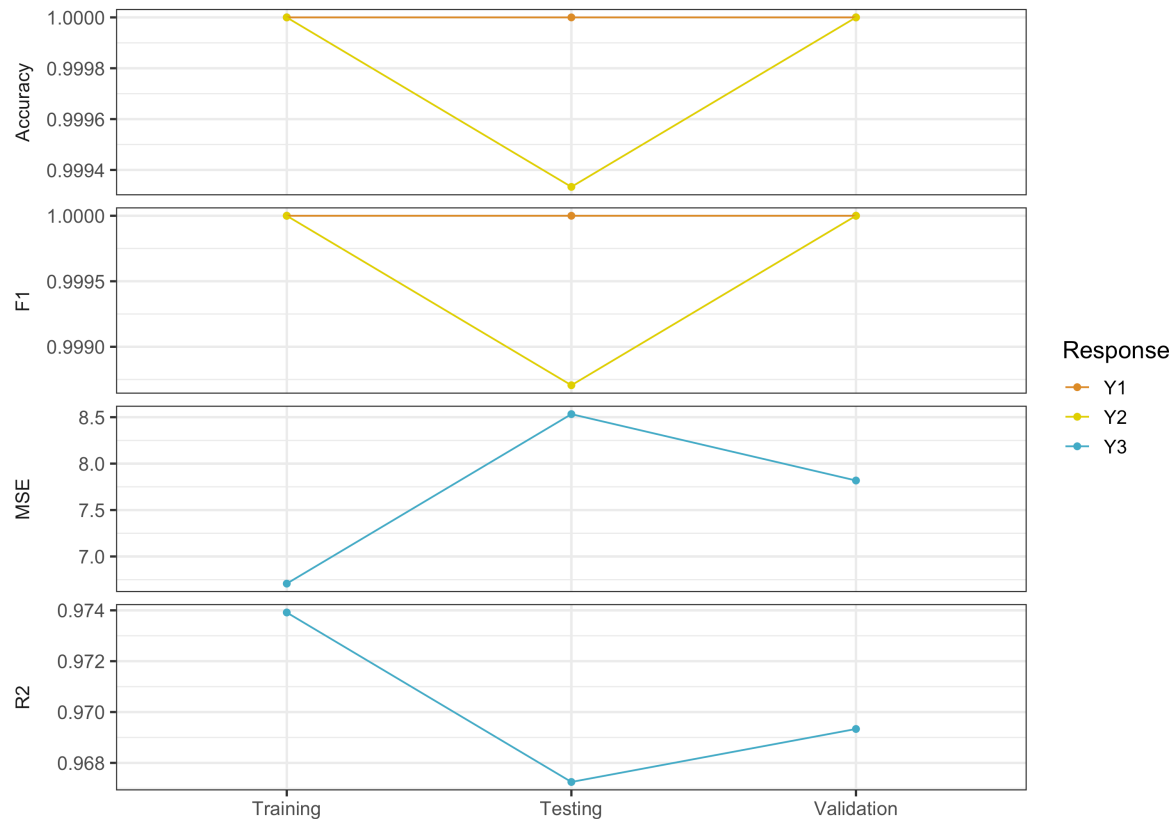
- Started with basic FPCA approach on the simple data
- Resulted in 1000 FPCs



- Y1 = True, Y2 = True
- Y1 = False, Y2 = True
- Y1 = True, Y2 = False
- Y1 = False, Y2 = False

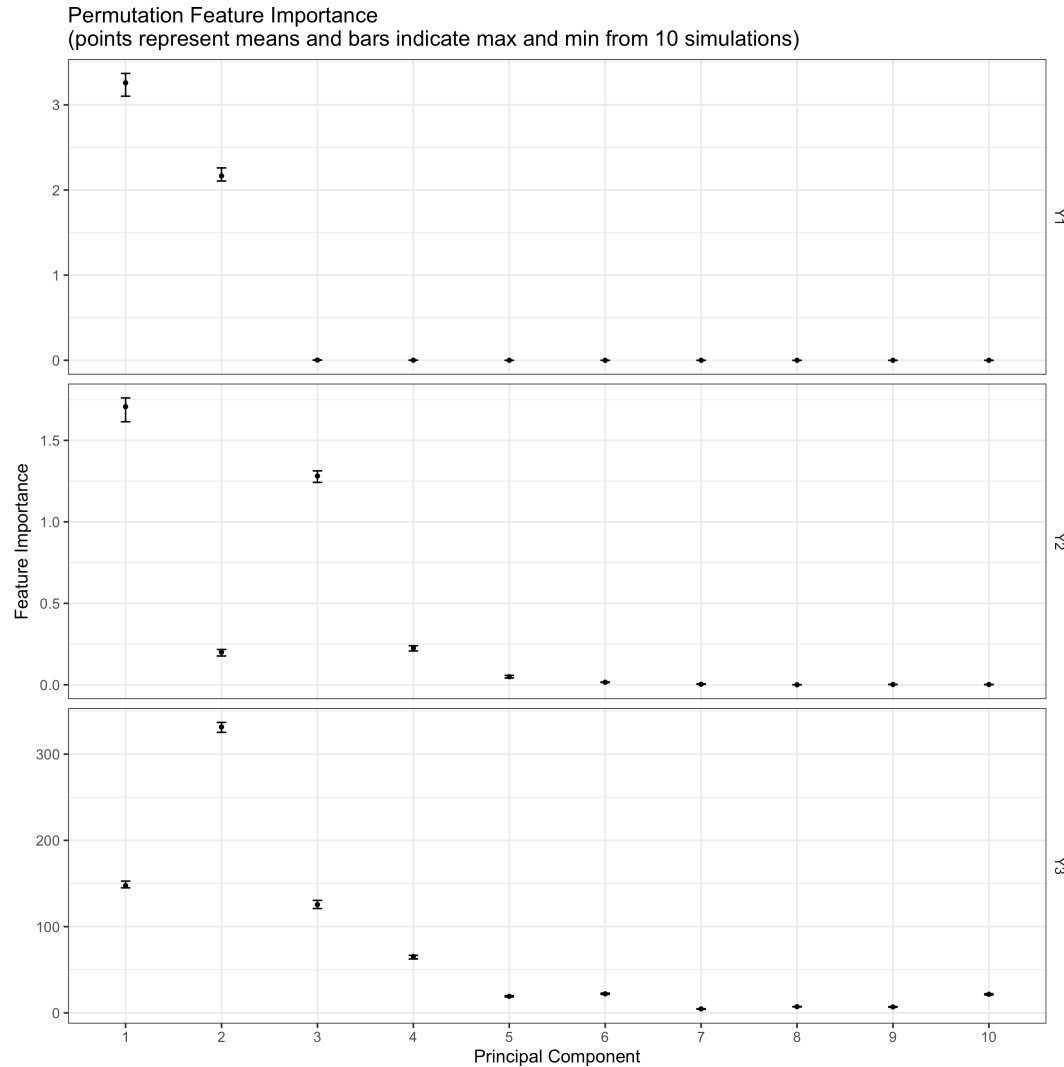
# Step 2: Fit neural network using PCs

- Three models (one per characteristic)
- 3 layers (50, 40, and 30 nodes)
- All 1000 PCs as feature





# Step 3: Apply permutation feature importance to determine important PCs

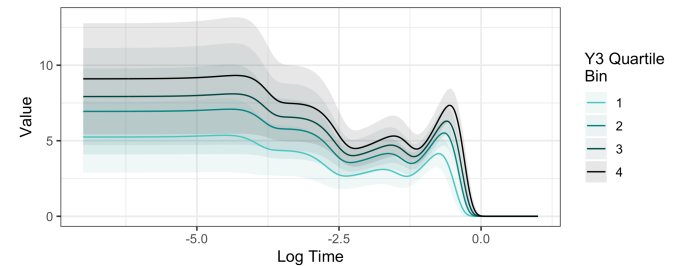
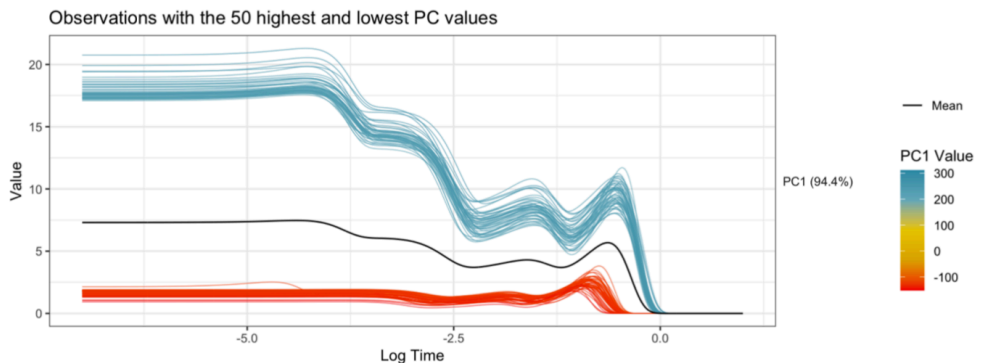
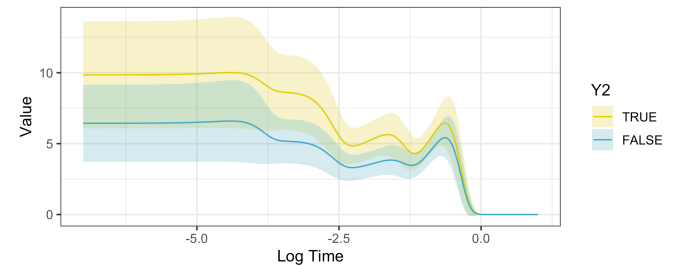
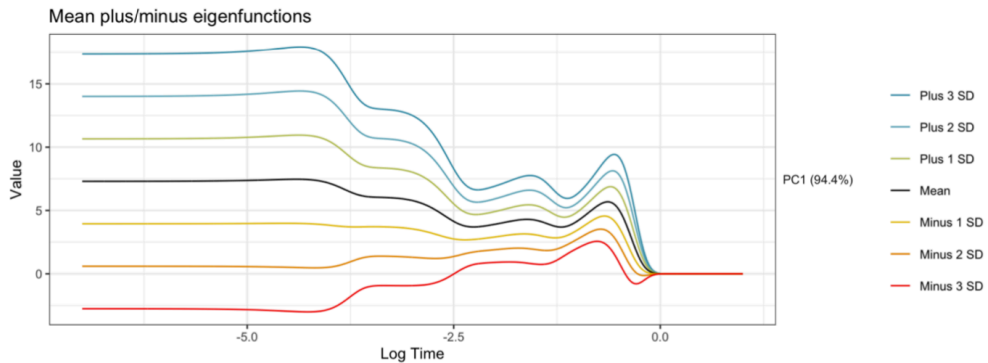
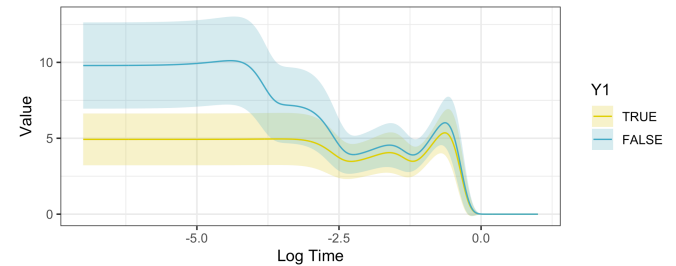
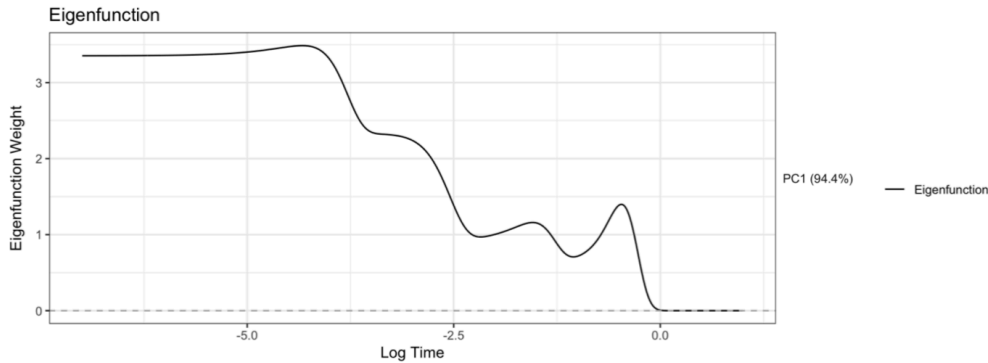


# Step 4: Interpret PCs using visualizations



## PC 1: Interpretation

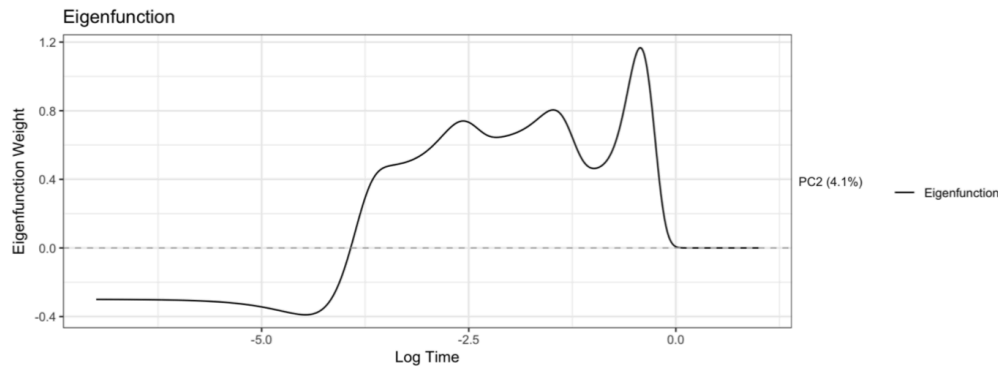
Important for Y1 and Y2  
Somewhat important for Y3



# Step 4: Interpret PCs using visualizations

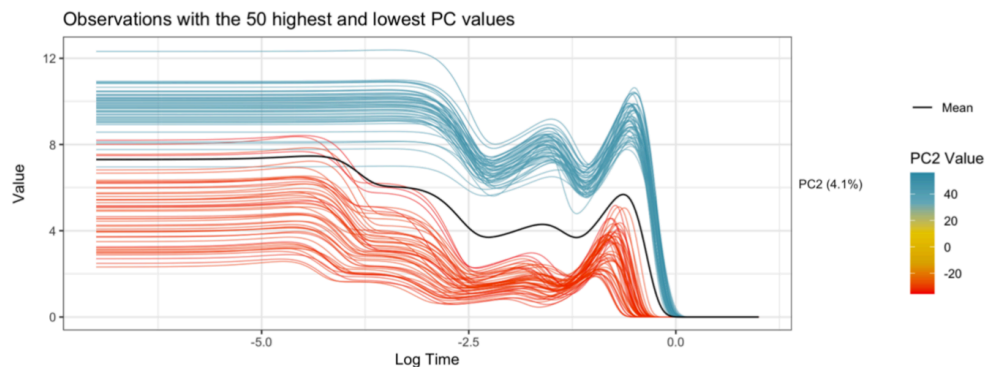
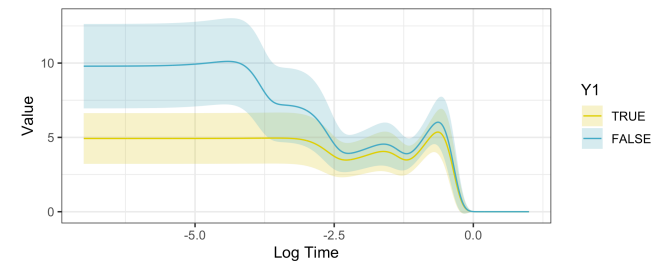
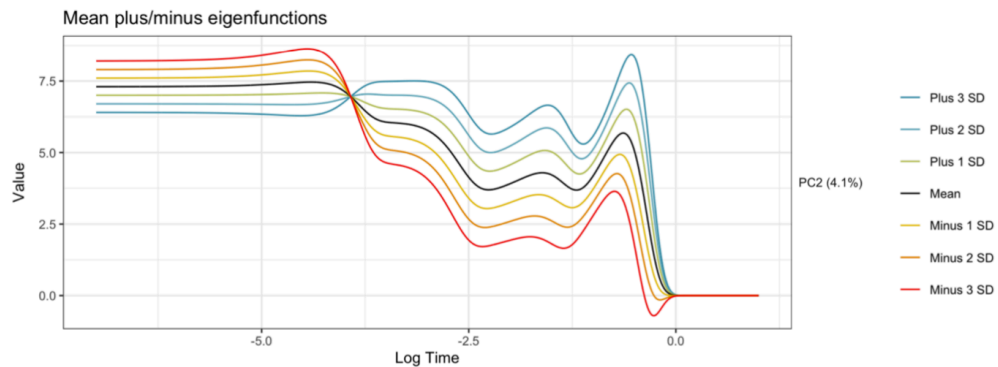
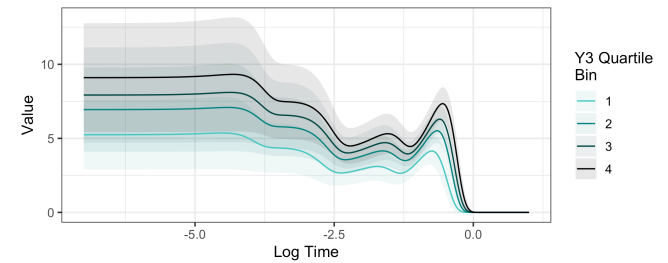


## PC 2: Interpretation



Important for Y3

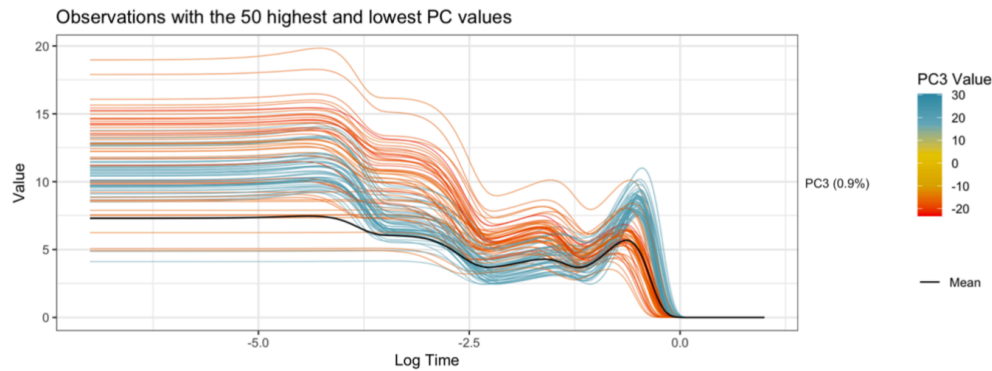
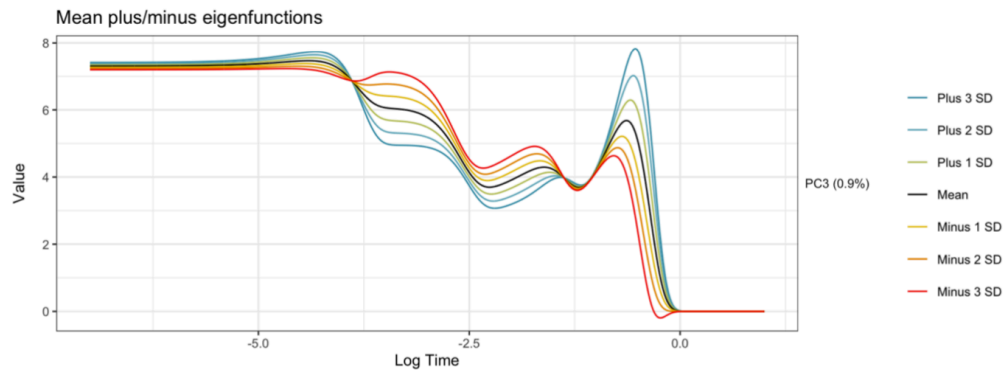
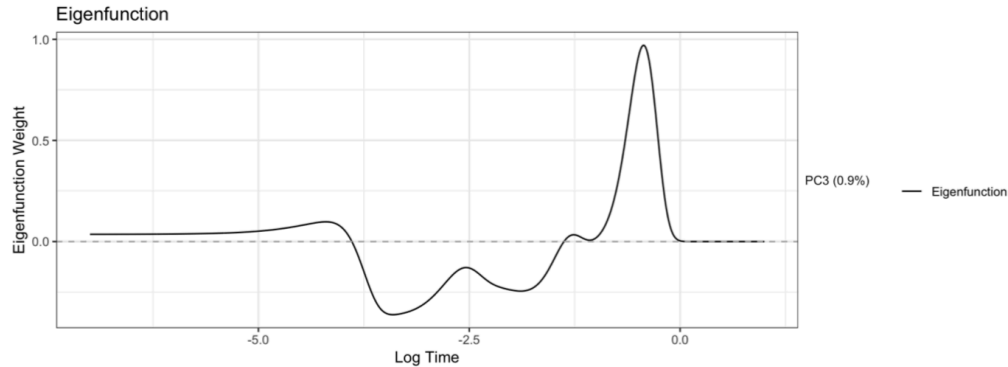
Somewhat important for Y1



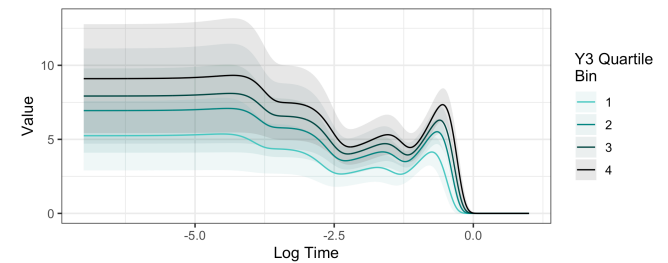
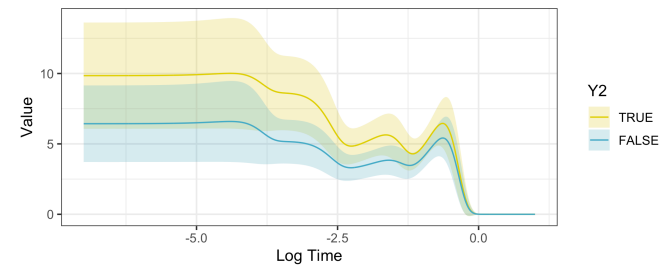
# Step 4: Interpret PCs using visualizations



## PC 3: Interpretation



Important for Y2  
Somewhat important for Y3





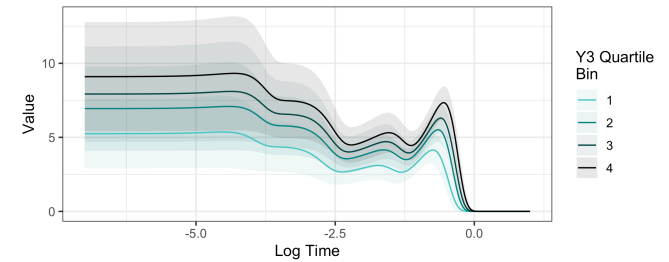
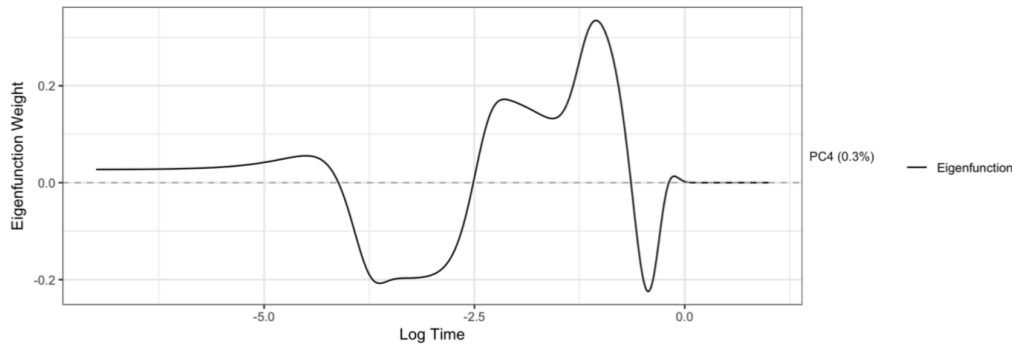
# Step 4: Interpret PCs using visualizations



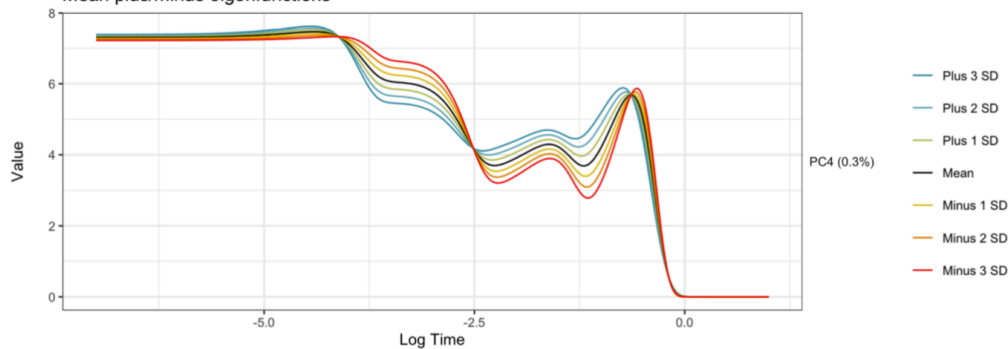
## PC 3: Interpretation

Somewhat important for Y3

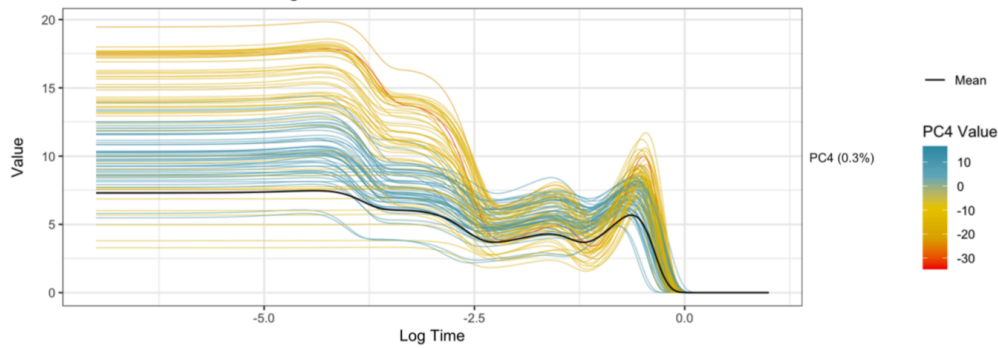
Eigenfunction



Mean plus/minus eigenfunctions



Observations with the 50 highest and lowest PC values



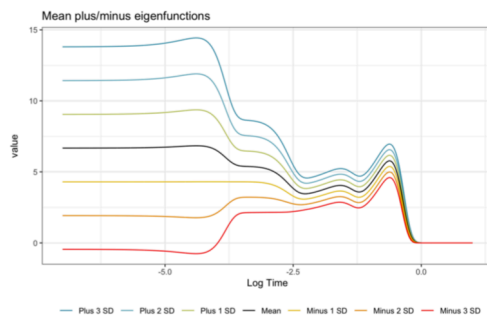


# Reflection on Interpretations with Simple Data

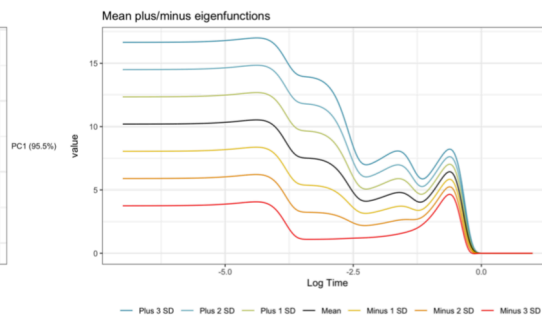
- For PCs 1 and 2, it is possible to use PFI to connect functional variability important to neural network predictions (in ways that made sense to SME)
- More difficult with PCs 3 and 4
- Some of the difficulty comes from the interactions between characteristics

PC 1 from data subset based on characteristics

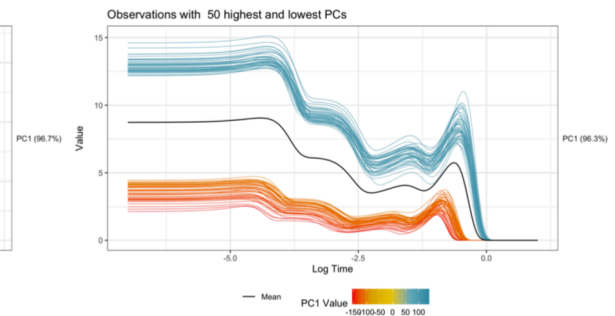
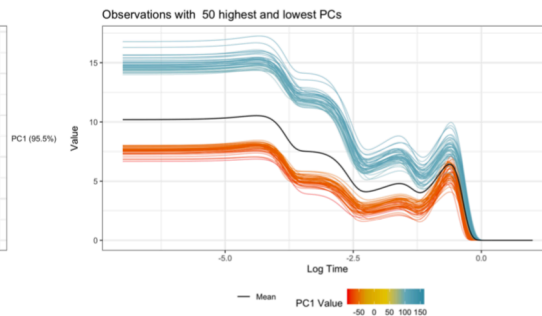
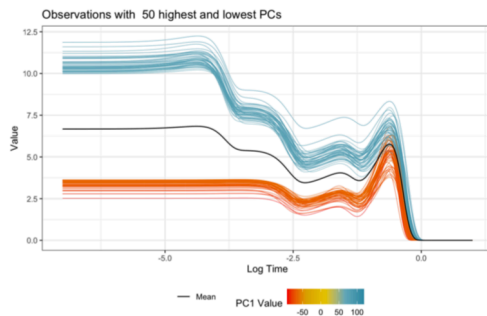
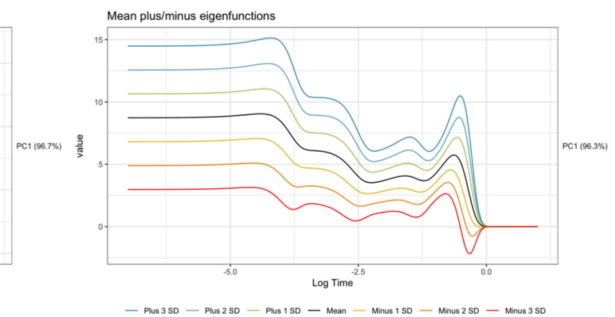
Focus on Y1



Focus on Y2



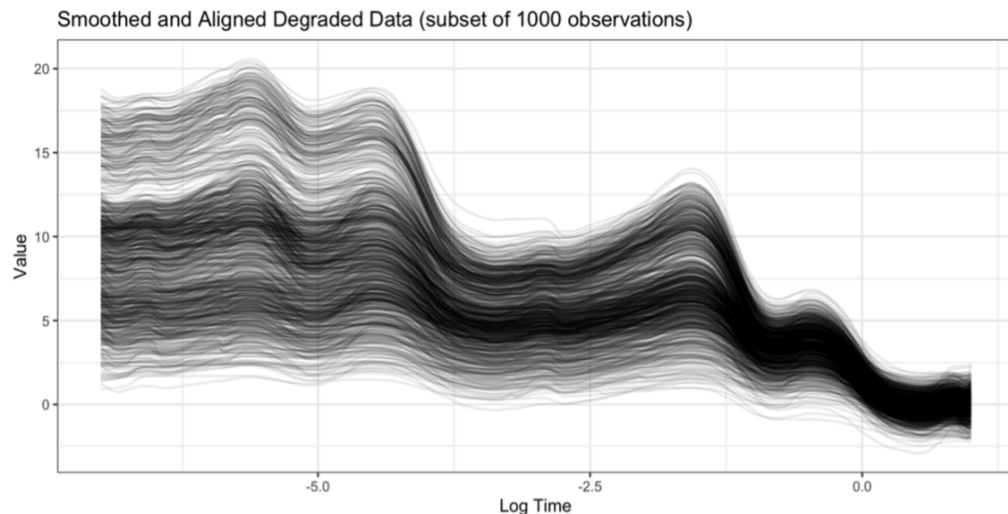
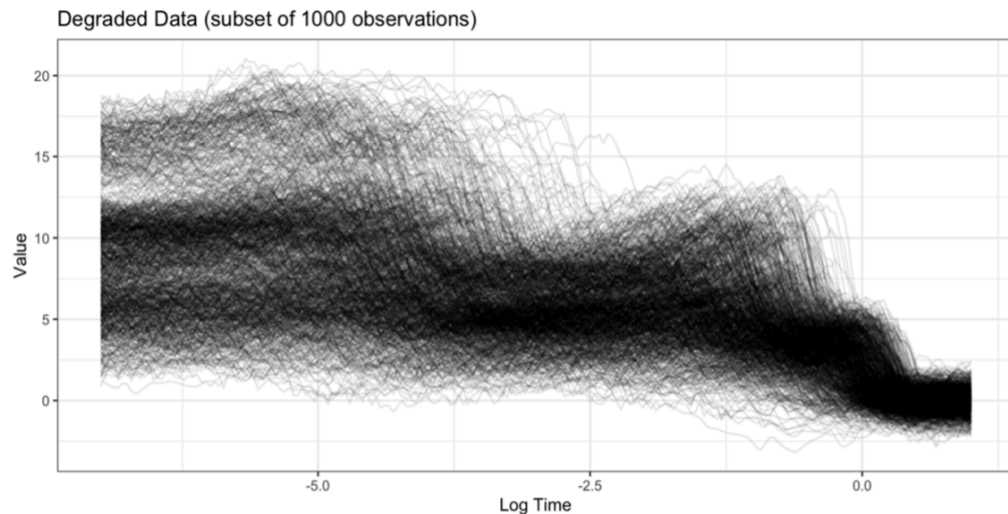
Focus on Y3



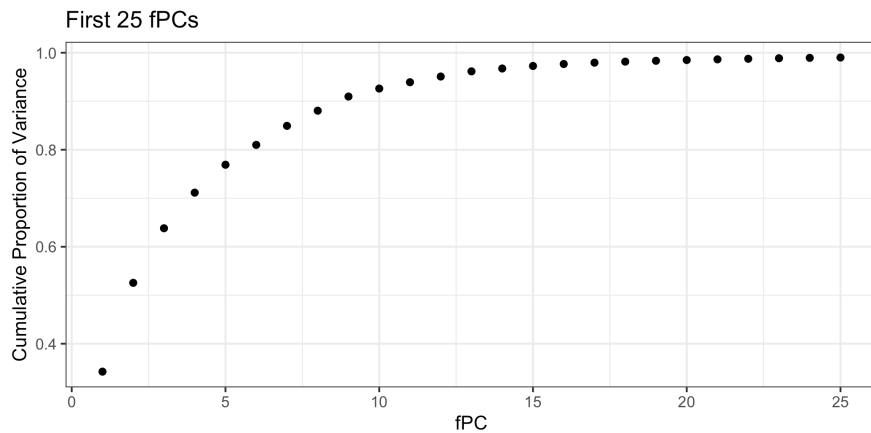
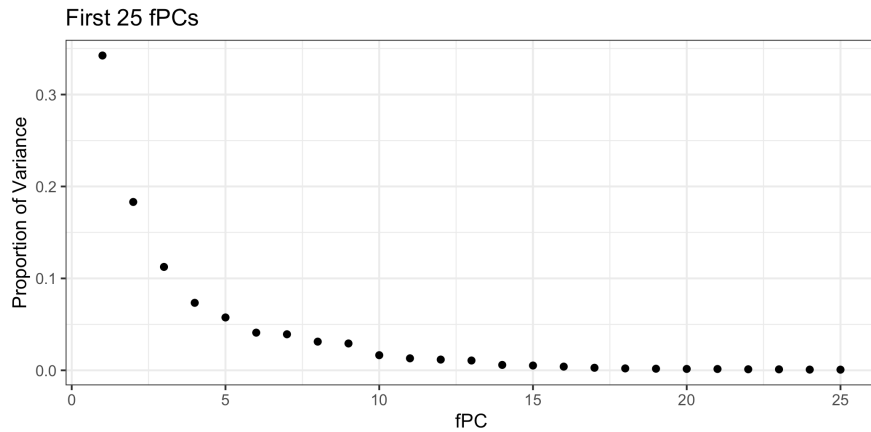
# Degraded Data and Joint fPCA



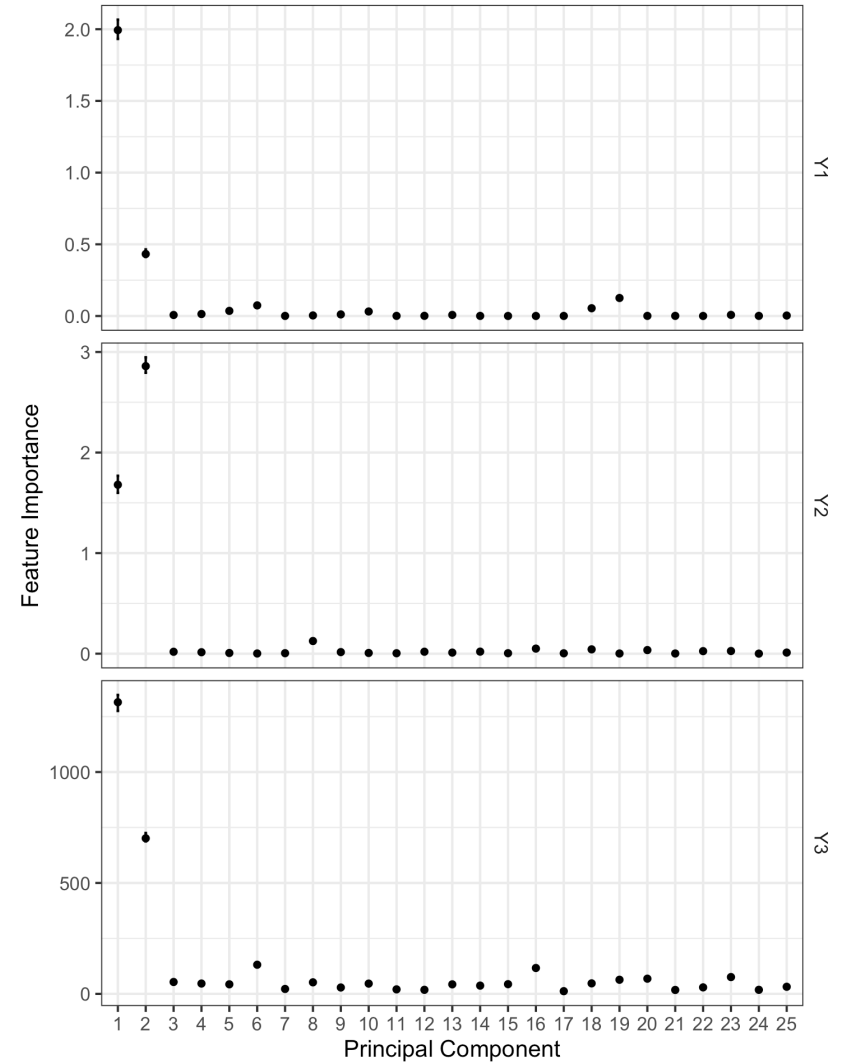
- Alignment done using time warping (fdasrvf package)
- Joint fPCA applied to the function to account for horizontal and vertical variability (fdasrvf package)
- Neural networks fit using the joint fPCs (one for each characteristic)
- PFI applied to identify important fPCs



# Joint FPCA Proportion of Variance and PFI

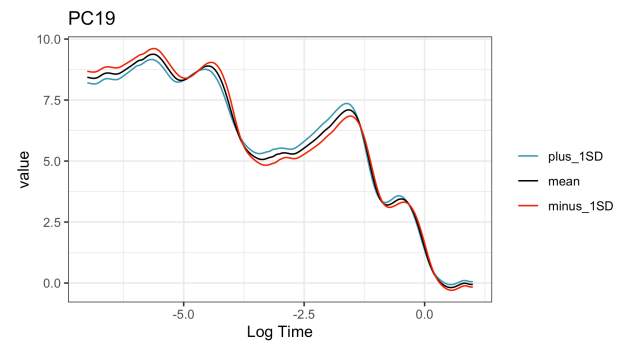
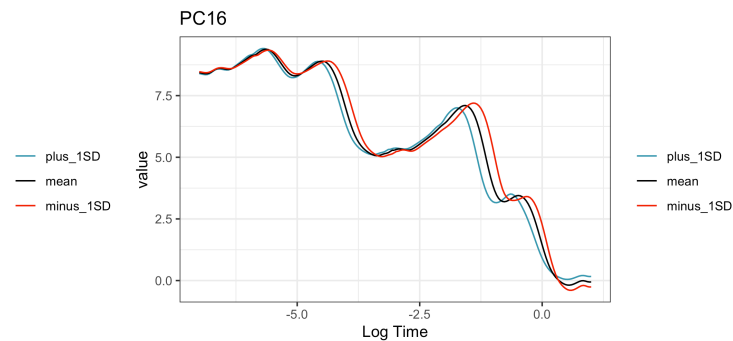
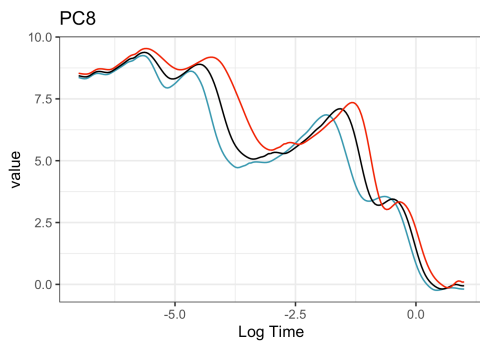
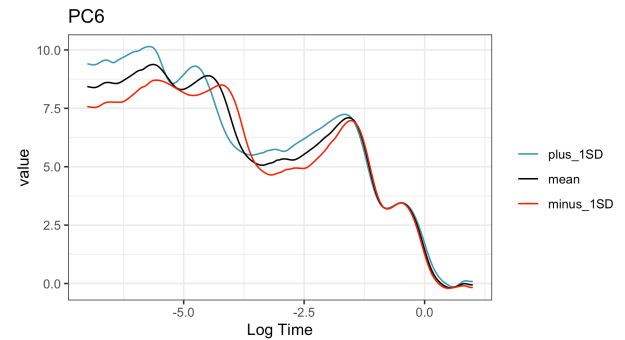
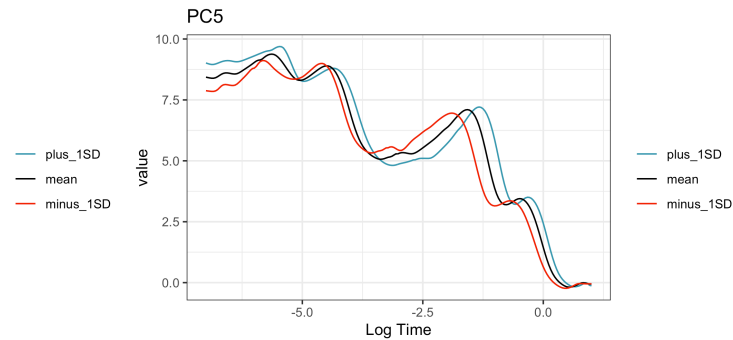
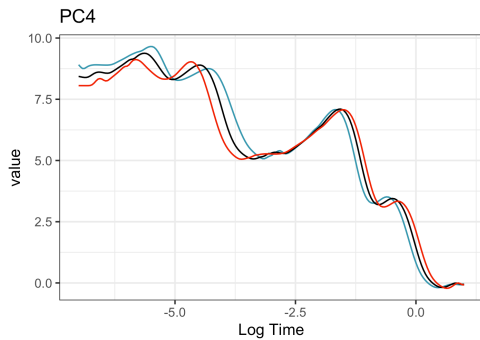
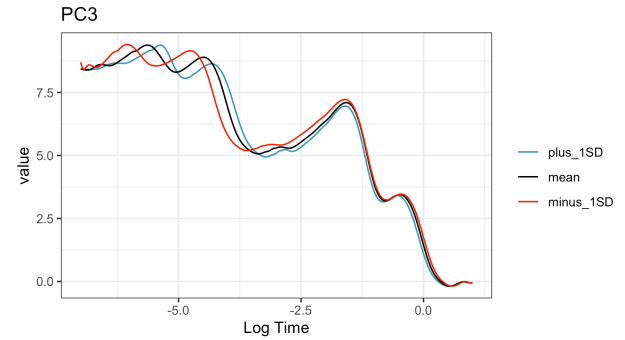
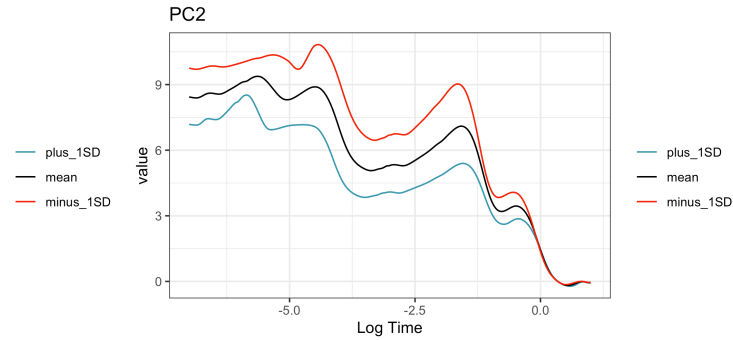
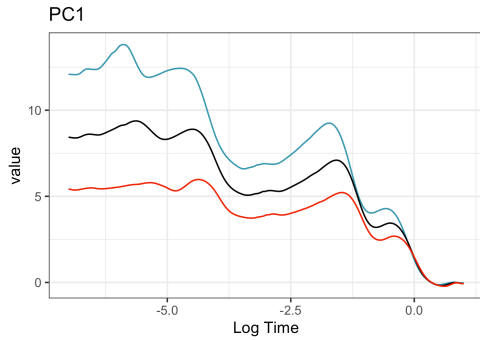


Permutation Feature Importance  
(points represent means and bars indicate max and min from 10 simulations)





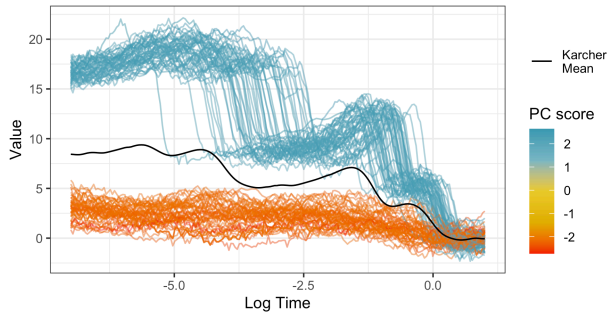
# PC Visualizations with Degraded Data



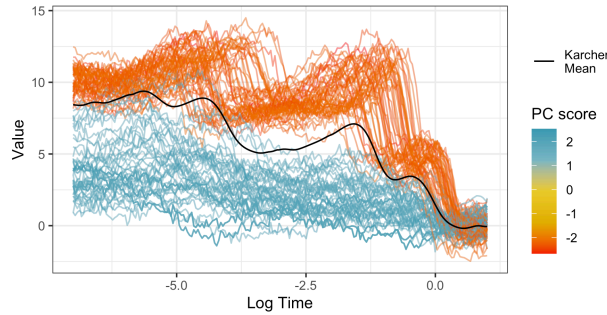


# PC Visualizations with Degraded Data

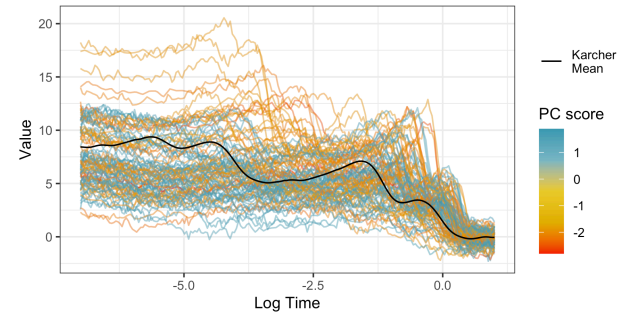
fPC 1



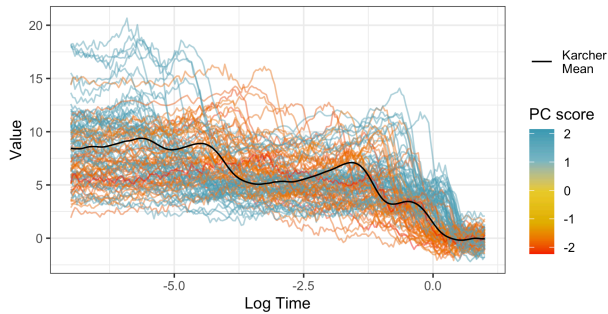
fPC 2



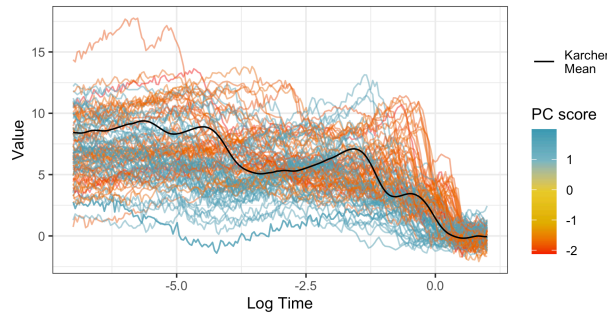
fPC 3



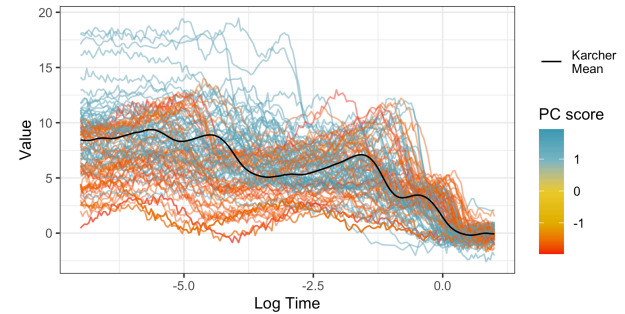
fPC 4



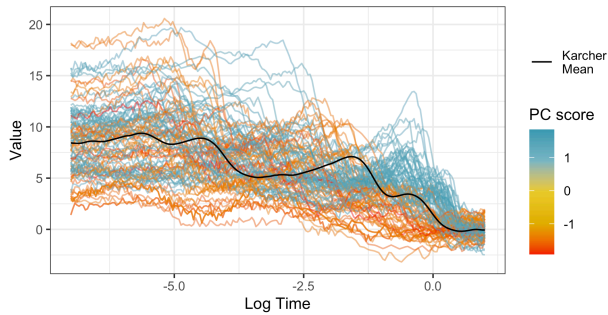
fPC 5



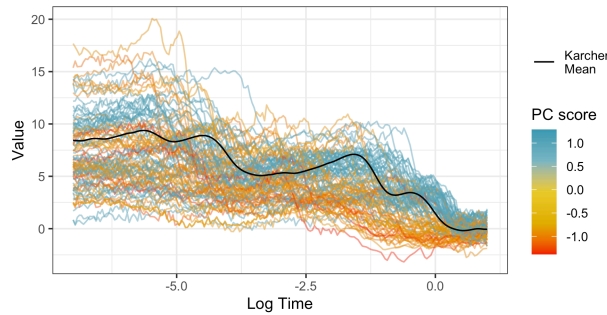
fPC 6



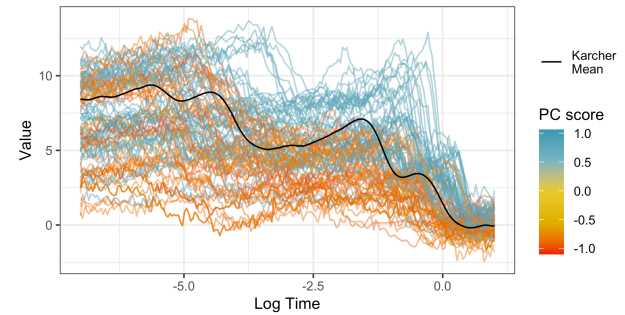
fPC 8



fPC 16



fPC 19





# Going Forward

Plans for future work

# Future Work

- Implementation on other datasets
- Application of varimax rotation (or other methods) to help with interpretation
- How to adjust PFI in the Bayesian setting?
- Simulation studies to better understand how correlation in functional data affects PFI if not accounted for





Questions?